Direct Strong H-infinity Norm Computation for SISO Time-Delay Systems

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Extended Abstract

The availability of robust methods to compute $\mathcal{H}_\infty$ norms is essential in a computer aided control system design [7]. The computation of $\mathcal{H}_\infty$ norm for finite dimensional plants is based on the relation between the intersections of the singular value curves corresponding to the transfer function with a given constant function and the existence of imaginary axis eigenvalues of a Hamiltonian matrix [4]. Based on this relation, the $\mathcal{H}_\infty$ norm can be computed via the well-known level set methods: a bisection based algorithm is described in [4] and quadratically converging algorithms in [1, 2]. The predictor-corrector algorithm for the $\mathcal{H}_\infty$ norm in [6] extends level set methods to a class of infinite-dimensional time-delay systems.

Recently in [5], we analyzed the properties of the $\mathcal{H}_\infty$ norm of general time-delay systems. We illustrated that the $\mathcal{H}_\infty$ norm may be sensitive with respect to arbitrarily small delay perturbations. Due to this sensitivity, we introduced the *strong $\mathcal{H}_\infty$ norm* which explicitly takes into account small delay perturbations, inevitable in any practical control application. A numerical algorithm to compute the strong $\mathcal{H}_\infty$ norm for time-delay systems based on a level set approach is also described in [5].

Level set based methods for computing $\mathcal{H}_\infty$ norms are inherently iterative, as they rely on the repeated computation of intersections of singular value curves with a constant function, whose value eventually converges to the $\mathcal{H}_\infty$ norm. In this work we describe an alternative numerical algorithm for computing the strong $\mathcal{H}_\infty$ norm of Single-Input-Single-Output (SISO) time-delay systems, which is not iterative, hence, the norm is directly computed in one step.

We consider stable SISO time-delay systems with the following transfer function representation,

$$G(s) = \left(C_0 + \sum_{i_c=1}^{n_c} C_{i_c} e^{-s\tau_{i_c}^c}\right)\left(sI - A_0 - \sum_{i_a=1}^{n_a} A_{i_a} e^{-s\tau_{i_a}^a}\right)^{-1}\left(B_0 + \sum_{i_b=1}^{n_b} B_{i_b} e^{-s\tau_{i_b}^b}\right) + \left(D_0 + \sum_{i_d=1}^{n_d} D_{i_d} e^{-s\tau_{i_d}^d}\right)$$

where upper case variables are real valued system matrices with appropriate dimensions and variables denoted by τ are system delays, nonnegative real numbers. By defining slack variables, we reformulate the transfer function of the SISO time-delay system as

$$G(s) = C\left(sE - \tilde{A}_0 - \sum_{i=1}^{n} \tilde{A}_i e^{-s\tau_i}\right)^{-1} B$$

where $\tilde{A}_i, i = 1, \ldots, n, B, C, D, E$ are real valued system matrices and $\tau_i, \quad i = 1, \ldots, n$ are system time-delays, nonnegative real numbers.

It is shown in [5] that the strong $\mathcal{H}_\infty$ norm of the transfer function is the maximum of the strong $\mathcal{H}_\infty$ norm of the so-called asymptotic transfer function of G and the $\mathcal{H}_\infty$ norm of G. We illustrate that the strong $\mathcal{H}_\infty$ norm of the asymptotic transfer function is equal to $\gamma_a = \sum_{i_d=0}^{n_d} |D_{i_d}|$. Subsequently, we show that the derivative of the $G(-s)G(s)$ w.r.t. s should be equal to zero at the local maximizers and minimizers of the curve $j\omega \mapsto \sigma_1(G(j\omega))$, including the frequency $s = j\omega_o$ where the $\mathcal{H}_\infty$ norm of G is achieved. Therefore we have to

compute the imaginary axis zeros of the transfer function

$$G'(-s)G(s) + G(-s)G'(s) \tag{1}$$

on an interval $\omega \in [0, \omega_{\max}]$ where $\omega_{\max}$ is the maximum frequency at which the singular value of the transfer function G is equal to γ_a, i.e., $\sigma(G(j\omega_{\max}) = \gamma_a$. We compute these imaginary axis zeros as solutions of an associated eigenvalue problem using spectral methods, [3]. If there is no such zero, then the strong $\mathcal{H}_\infty$ norm of G is equal to γ_a, otherwise it is equal to the maximum of the transfer function over the computed imaginary axis zeros.

In contrast to the existing numerical algorithm for time-delay systems [6], the algorithm is not an iterative but a direct method, as it relies on the direct computation of the extrema in the singular values curve corresponding to the transfer function.

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