Sensitivity Minimization by Stable Controllers: An Interpolation Approach for Suboptimal Solutions

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***Abstract*— Weighted sensitivity minimization is studied within the framework of strongly stabilizing (stable) $\mathcal{H}^\infty$ controller design for a class of infinite dimensional systems. Earlier results show that the optimal solution can be obtained using Nevanlinna-Pick interpolation. However, in this approach the controller turns out to be non-causal: it contains a time-advance. In this note, by putting an extra norm condition in the problem formulation causal suboptimal controllers are obtained using the same interpolation approach.**

I. Introduction

In this note we study sensitivity minimization problem for a class of infinite dimensional systems. The goal is to minimize the $\mathcal{H}^\infty$ norm of the weighted sensitivity by using stable controllers from the set of all stabilizing controllers for the given plant. This problem is a special case of strongly stabilizing (i.e. stable) controller design studied earlier, see for example [2], [3], [4], [5], [9], [12], [13], [15], [17], [18], [19], [20], [24], [26], [27], and their references, for different versions of the problem. The methods in [1], [7] give optimal (sensitivity minimizing) stable $\mathcal{H}^\infty$ controllers for finite dimensional SISO plants. Other methods provide sufficient conditions to find stable suboptimal $\mathcal{H}^\infty$ controllers. As far as infinite dimensional systems are concerned, [8], [22] considered systems with time delays. Most recently, [11] gave an extension of the technique used in [7] to a class of time delay systems.

It has been observed that (see e.g. [7], [11]) the Nevanlinna-Pick interpolation approach used in these papers lead to stable controllers with "essential singularity" at infinity. This means that the controller is non-causal, i.e. it contains a time advance, as seen in the examples. In this note, by putting a norm bound condition on the inverse of the weighted sensitivity we obtain causal suboptimal controllers using the same interpolation approach. This extra condition also gives an upper bound on the $\mathcal{H}^\infty$ norm of the stable controller to be designed.

The problem definition and a summary of the results from [11] are given in Section II. Construction procedure for causal (sub)optimal strongly stabilizing $\mathcal{H}^\infty$ controllers appears in Section III. Examples, involving time delay systems with finitely many zeros in the right half plane, can be found in Section IV. Concluding remarks are made in Section V.

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II. Problem Definition and Preliminary Results

Consider the standard unity feedback system with single-input-single-output plant P and controller C. The sensitivity function for this feedback system is $S = (1+PC)^{-1}$. We say that the controller stabilizes the plant if S, CS and PS are in $\mathcal{H}^\infty$. The set of all stabilizing controllers for a given plant P is denoted by $\mathcal{S}(P)$, and we define $\mathcal{S}_\infty(P) = \mathcal{S}(P) \cap \mathcal{H}^\infty$ as the set of all strongly stabilizing controllers.

For a given minimum phase filter $W(s)$ the classical weighted sensitivity minimization problem is to find

$$\gamma_{\text{o}} = \inf_{C \in \mathcal{S}(P)} \|W(1+PC)^{-1}\|_\infty .$$

When we restrict the controller to the set $\mathcal{S}_\infty(P)$ we have the problem of weighted sensitivity minimization by a stable controller (WSMSC): in this case the goal is to find

$$\gamma_{\text{ss}} = \inf_{C \in \mathcal{S}_\infty(P)} \|W(1+PC)^{-1}\|_\infty , \tag{1}$$

and the corresponding optimal controller $C_{\text{ss,opt}} \in \mathcal{S}_\infty(P)$.

The class of plants to be considered here are in the form

$$P(s) = \frac{M_n(s)}{M_d(s)} N_o(s) \tag{2}$$

where M_n, M_d are inner and N_o is outer with relative degree equal to $n_o \in \mathbb{N}$. We will assume that M_n is rational (finite Blaschke product), but M_d and N_o can be infinite dimensional. A typical example of such plants are retarded or neutral time delay systems written in the form

$$P(s) = \frac{R(s)}{T(s)} = \frac{\sum_{i=1}^{n_r} R_i(s) e^{-h_i s}}{\sum_{j=1}^{n_t} T_j(s) e^{-\tau_j s}} \tag{3}$$

where

(i) R_i and T_j are stable, proper, finite dimensional transfer functions, $i = 1, \ldots, n_r$, and $j = 1, \ldots, n_t$;

(ii) R and T have no imaginary axis zeros, but they may have finitely many zeros in $\mathbb{C}_+$;[1]

(iii) time delays, h_i and τ_j are rational numbers such that $0 = h_1 < h_2 < \ldots < h_{n_r}$, and $0 = \tau_1 < \tau_2 < \ldots < \tau_{n_t}$.

We refer to [10] for details on how to put the time delay system (3) into general form (2).

Let $s_1, \ldots, s_n$ be the zeros of $M_n(s)$ in $\mathbb{C}_+$. Then, WSMSC problem can be solved by constructing a unit[2]

[1] Assumption on T can be relaxed to allow infinitely many zeros in $\mathbb{C}_+$, see [10] for details.

[2] A function $F \in \mathcal{H}^\infty$ is called a unit if $F^{-1} \in \mathcal{H}^\infty$.

$F \in \mathcal{H}^\infty$ satisfying $\|F\|_\infty \leq 1$, and

$$F(s_i) = \frac{W(s_i)}{\gamma M_d(s_i)} =: \frac{\omega_i}{\gamma}, \quad i = 1, \ldots, n. \tag{4}$$

Once such an F is found the controller

$$C_\gamma(s) = \frac{W(s) - \gamma M_d(s)F(s)}{\gamma M_n(s)F(s)} \; \frac{N_o(s)^{-1}}{(1+\varepsilon s)^{n_o}} \;, \quad \varepsilon \searrow 0 \tag{5}$$

is in $\mathcal{S}_\infty(P)$ and it leads to $\|W(1+PC)^{-1}\|_\infty \leq \gamma$, see e.g. [7], [11]. Therefore, $\gamma_{\rm ss}$ is the smallest γ for which there exists $F \in \mathcal{H}^\infty$, such that $F^{-1} \in \mathcal{H}^\infty$, $\|F\|_\infty \leq 1$, and the interpolation conditions (4) hold.

This problem has been solved by using the Nevanlinna-Pick interpolation as follows. First define

$$G(s) = -\ln F(s) \qquad F(s) = e^{-G(s)}. \tag{6}$$

Now, we want to find an analytic function $G : \mathbb{C}_+ \to \mathbb{C}_+$ such that

$$G(s_i) = -\ln \omega_i + \ln \gamma - j2\pi \ell_i =: \nu_i, \quad i = 1, \ldots, n \tag{7}$$

where ℓ_i is a free integer due to non-uniqueness of the complex logarithm. Note that when $\|F\|_\infty \leq 1$ the function G has a positive real part hence it maps $\mathbb{C}_+$ into $\mathbb{C}_+$. Let $\mathbb{D}$ denote the open unit disc, and transform the problem data from $\mathbb{C}_+$ to $\mathbb{D}$ by using a one-to-one conformal map $z = \phi(s)$. The transformed interpolation conditions are

$$f(z_i) = \frac{\omega_i}{\gamma}, \quad i = 1, \ldots, n \tag{8}$$

where $z_i = \phi(s_i)$ and $f(z) = F(\phi^{-1}(z))$. The transformed interpolation problem is to find a unit with $\|f\|_\infty \leq 1$ such that interpolation conditions (8) are satisfied. By the transformation $g(z) = -\ln f(z)$, the interpolation problem can be written as,

$$g(z_i) = \nu_i, \quad i = 1, \ldots, n.$$

Define $\phi(\nu_i) =: \zeta_i$. If we can find an analytic function $\tilde{g} : \mathbb{D} \to \mathbb{D}$, satisfying

$$\tilde{g}(z_i) = \zeta_i \qquad i = 1, \ldots, n$$

then the desired $g(z)$, hence $f(z)$ and $F(s)$ can be constructed from $g(z) = \phi^{-1}(\tilde{g}(z))$. The problem of finding such $\tilde{g}$ is the well-known Nevanlinna-Pick problem, [6], [14], [25]. The condition for the existence of an appropriate g can be given directly: there exists such an analytic function $g : \mathbb{D} \to \mathbb{C}_+$ if and only if the Pick matrix $\mathcal{P}$,

$$\mathcal{P}(\gamma, \{\ell_i\})_{i,k} = \left[\frac{2\ln\gamma - \ln\omega_i - \ln\bar{\omega}_k + j2\pi\ell_{k,i}}{1 - z_i \bar{z}_k} \right] \tag{9}$$

is positive semi-definite, where $\ell_{k,i} = \ell_k - \ell_i$ are integers. In [7], it is mentioned that the possible integer sets $\{\ell_i\}$ are finite and in all possible integer sets $\{\ell_i\}_k$, $k = 1, \ldots, r$, there exists a minimum value, $\gamma_{\rm ss}$, such that $\mathcal{P}(\gamma_{\rm ss}, \{\ell_i\}_k) \geq 0$.

The Nevanlinna-Pick problem posed above can be solved as outlined in [6], [14], [25]. As noted in [7], [11] and we illustrate with an example in Section IV, in general, as γ decreases to $\gamma_{\rm ss}$ the function $G(s)$ takes the form

$$G(s) \approx k_\gamma \frac{s}{1+\delta s} \;, \quad \text{where} \quad k_\gamma \in \mathbb{R}_+ \text{ and } \; \delta \searrow 0.$$

Therefore, in the optimal case $F(s)$ has an essential singularity at infinity, i.e. it is in the form $F(s) = e^{-k_\gamma s}$, thus $F^{-1} \notin \mathcal{H}^\infty$. Clearly, this violates one of the design conditions and leads to a non-causal controller (5), which contains a time advance. In the next section to circumvent this problem we propose to put an $\mathcal{H}^\infty$ norm bound on F^{-1}.

III. Modified Interpolation Problem

The controller (5) gives the following weighted sensitivity

$$W(s)(1 + P(s)C_\gamma(s))^{-1} = \gamma M_d(s)F(s) \tag{10}$$

where $F, F^{-1} \in \mathcal{H}^\infty$, $\|F\|_\infty \leq 1$ and (4) holds. Since one of the conditions on F is to have $F^{-1} \in \mathcal{H}^\infty$ it is natural to consider a norm bound

$$\|F^{-1}\|_\infty \leq \rho \tag{11}$$

for some fixed $\rho > 1$. This also puts a bound on the $\mathcal{H}^\infty$ norm of the controller; more precisely, we have

$$\|C_\gamma\|_\infty \leq \|N_o\|_\infty^{-1} \left(1 + \frac{\rho}{\gamma}\|W\|_\infty\right).$$

Recall that we are looking for an F in the form $F(s) = e^{-G(s)}$, for some analytic $G : \mathbb{C}_+ \to \mathbb{C}_+$ satisfying $G(s_i) = \nu_i$, $i = 1, \ldots, n$. In this case we will have $|F(s)| = |e^{-{\rm Re}(G(s))}| \leq 1$ for all $s \in \mathbb{C}_+$. On the other hand, $F^{-1}(s) = e^{G(s)}$. Thus, in order to satisfy (11), G should have a bounded real part, namely

$$0 < {\rm Re}(G(s)) < \ln(\rho) =: \sigma_o$$

Accordingly, define $\mathbb{C}_+^{\sigma_o} := \{s \in \mathbb{C}_+ : 0 < {\rm Re}(s) < \sigma_o\}$. Then, the analytic function G we construct should take $\mathbb{C}_+$ into $\mathbb{C}_+^{\sigma_o}$. Note from (7) that in order for this modified problem to make sense γ should not be too large (or ρ should be large enough) so that we have a feasible interpolation data, i.e. $\nu_i \in \mathbb{C}_+^{\sigma_o}$. Now take a conformal map $\psi : \mathbb{C}_+^{\sigma_o} \to \mathbb{D}$, and set $\zeta_i := \psi(\nu_i)$, $z_i = \phi(s_i)$, where as before ϕ is a conformal map from $\mathbb{C}_+$ to $\mathbb{D}$. Then, the problem is again transformed to a Nevanlinna-Pick interpolation: find an analytic function $\tilde{g} : \mathbb{D} \to \mathbb{D}$ such that $\tilde{g}(z_i) = \zeta_i$, $i = 1, \ldots n$. Once $\tilde{g}$ is obtained, the function G is determined as $G(s) = \psi^{-1}(\tilde{g}(\phi(s)))$. Typically, we take

$$\begin{aligned} z = \phi(s) \;&=\; \frac{s-1}{s+1} \qquad s = \phi^{-1}(z) = \frac{1+z}{1-z} \\ z = \psi(s) \;&=\; \frac{je^{-j\pi s/\sigma_o} - 1}{je^{-j\pi s/\sigma_o} + 1} \\ s = \psi^{-1}(z) \;&=\; \frac{\sigma_o}{\pi}\left(\frac{\pi}{2} + j\ln(\frac{1+z}{1-z})\right), \end{aligned} \tag{12}$$

see e.g. [16].

It is interesting to note that in this modified problem $\gamma_{\rm ss}$ (smallest γ for which a feasible $\tilde{g}$ exists) depends on ρ. As ρ decreases we expect that $\gamma_{\rm ss}$ will increase; and as $\rho \to \infty$ we expect that γ_{ss} will converge to the value found from the unrestricted interpolation problem summarized in Section II.

IV. Example

Consider a plant with infinitely many poles in $\mathbb{C}_+$:

$$P(s) = \frac{(s+1)+4e^{-3s}}{(s+1)+2(s-1)e^{-2s}} \quad (13)$$

$$= \frac{R(s)}{T(s)} = \frac{1e^{-0s} + \left(\frac{4}{s+1}\right)e^{-3s}}{1e^{-0s} + \left(\frac{2s-2}{s+1}\right)e^{-2s}}.$$

It can be shown that R has only two $\mathbb{C}_+$ zeros at $s_{1,2} \approx 0.3125 \pm 0.8548j$. Also, T has infinitely many $\mathbb{C}_+$ zeros converging to $\ln\sqrt{2} \pm j(k+\frac{1}{2})\pi$ as $k \to \infty$. As shown in [11], this plant can be re-written as (2) with

$$M_n(s) = \frac{(s-s_1)(s-s_2)}{(s+s_1)(s+s_2)}, \quad M_d(s) = \frac{T(s)}{\bar{T}(s)},$$

$$N_o(s) = \frac{R(s)}{M_n(s)}\frac{1}{\bar{T}(s)}.$$

where $\bar{T}(s) = e^{-2s}T(-s)\left(\frac{s-1}{s+1}\right) = 2 + \left(\frac{s-1}{s+1}\right)e^{-2s}$.

Let the weighting function be given as

$$W(s) = \frac{1+0.1s}{s+1}.$$

Then, the interpolation conditions are $\omega_{1,2} = 0.79 \mp 0.42j$. Applying the procedure of [11], summarized in Section II, we find $\gamma_{ss} = 1.0704$. The optimal interpolating function is

$$F(s) = e^{-0.57s} \quad (14)$$

and hence the optimal controller is written as

$$C_{\gamma_{ss}} = \frac{\frac{1+0.1s}{s+1} - 1.0704\left(\frac{s+1+2(s-1)e^{-2s}}{2(s+1)+(s-1)e^{-2s}}\right)e^{-0.57s}}{1.0704\left(\frac{s+1+4e^{-3s}}{2(s+1)+(s-1)e^{-2s}}\right)e^{-0.57s}}.$$

Clearly, $F^{-1} \notin \mathcal{H}^\infty$ and the controller is non-causal, it includes a time advance $e^{+0.57s}$.

If we now apply the modified interpolation idea we see that as $\rho \to \infty$ the smallest γ for which the problem is solvable, i.e. γ_{ss}, approaches to 1.0704, which is the optimal performance level found earlier. On the other hand, as ρ decreases γ_{ss} increases, and there is a minimum value of $\rho = e^{0.88} = 2.41$, below which there is no solution to the interpolation problem. See Figure 1.

For $\sigma_o = 3$, i.e. $\rho = e^3 = 20$, we have $\gamma_{ss}(\rho) = 1.08$, and the resulting interpolant is given by

$$\tilde{G}(s) := \tilde{g}(\phi(s)) = j\frac{-0.99794(s-3.415)(s+1)}{(s+3.406)(s+1.001)}.$$

The optimal $F(s) = e^{-G(s)}$ is determined from

$$G(s) = \psi^{-1}(\tilde{G}(s))$$

where ψ^{-1} is as defined in (12). The optimal F is

$$F(s) = \exp\left(-\frac{\sigma_o}{2} - \frac{j\sigma_o}{\pi}\ln\left(\frac{1+\tilde{G}(s)}{1-\tilde{G}(s)}\right)\right). \quad (15)$$

Note that $\tilde{G}(s) \approx j\frac{\pi-s}{\pi+s}$, if we use this function, $\tilde{G}(s)$, then the optimal $F(s)$ is infinite dimensional with $F(\infty) = 1$,

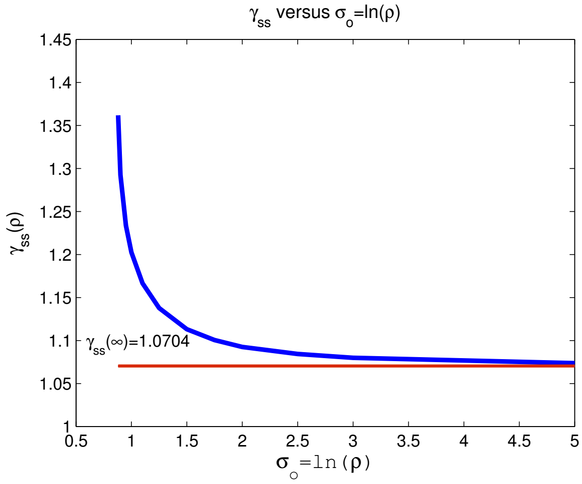

Fig. 1. γ_{ss} versus $\rho = e^{\sigma_o}$

i.e. $F^{-1}(\infty) = 1$ is well defined, as opposed to the optimal solution (14). Moreover for this $\tilde{G}$, magnitude of F on the Im-axis is piecewise constant

$$|F(j\omega)| = \begin{cases} 1 & \text{if} \quad 0 \le \omega < \pi \\ e^{-\sigma_o} = 0.05 & \text{if} \quad \omega > \pi \end{cases}$$

and its phase is

$$\angle F(j\omega) = -\frac{\sigma_o}{\pi}\ln\left|\frac{\pi+\omega}{\pi-\omega}\right| \quad \text{for} \quad \omega \ne \pi.$$

In order to get finite dimensional controllers we need to approximate $F(s)$. However, approximation of $F(s)$ given in (15) requires very high order rational functions.

Another way to obtain finite dimensional interpolating function $F(s)$ is to search for a proper free parameter in the set of all suboptimal solutions to the interpolation problem of finding $F \in \mathcal{H}^\infty$ satisfying (4), so that $F^{-1} \in \mathcal{H}^\infty$. For a given $\gamma > \gamma_{ss}$ we can parameterize all suboptimal solutions to this problem as, (see e.g. [6])

$$f(z) = \frac{\tilde{P}(z)q(z) + \tilde{Q}(z)}{P(z) + Q(z)q(z)}, \quad \|q\|_\infty \le 1, \quad (16)$$

where $\tilde{P}, \tilde{Q}, P, Q$ are computed as in [6], [14], [25]. Using first-order free parameter

$$q(z) = \frac{az+b}{z+c},$$

we search for a unit f in the set determined by (16). Since $\|q\|_\infty \le 1$, the parameters (a,b,c) are in the set

$$\mathcal{D}_q := \{(a,b,c) : |c| \ge 1, \ |a+b| \le |c+1|, \ |a-b| \le |c-1|\}.$$

Then a unit function f can be found if there exist $(a,b,c) \in \mathcal{D}_q$ such that

$$(az+b)\tilde{P}(z) + (z+c)\tilde{Q}(z) \quad (17)$$

has no zeros in $\mathbb{D}$. The problem of finding (a,b,c) such that (17) has no zeros in $\mathbb{D}$ is equivalent to stabilization of discrete-time systems by first-order controllers considered in

[23]. So we take the intersection of the parameters found using [23] and the set $\mathcal{D}_q$. The stabilization set (a, b, c) is determined by fixing c and obtaining the stabilization set in $a - b$ plane by stability boundaries in D-decomposition.

For the above example, let $\gamma = 1.2 > 1.07 = \gamma_{ss}$. After the calculation of $\tilde{P}$, $\tilde{Q}$, P, Q, for each fixed c we obtain feasible parameter pairs (a, b) resulting in a unit $f(z)$ as shown in Figure 2. Note that all values in (a, b, c) parameter set results in stable suboptimal $\mathcal{H}^\infty$ controller which gives flexibility in design to meet other design requirements.

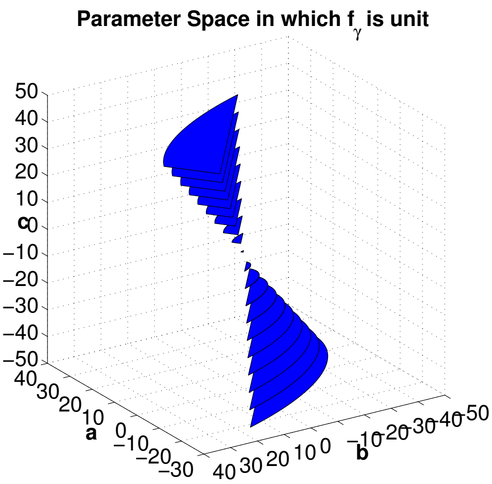

Fig. 2. Feasible (a, b, c) for f to be a unit.

In Figure 3, stability region for (17) is given for $c = 30$. Red and blue lines are real and complex-root crossing boundaries respectively. The yellow colored region (labeled as region 0 in the grayscale print) is where the polynomial (17) has no $\mathbb{C}_+$ zero and the corresponding $\mathcal{H}^\infty$ controller is stable. The value of $\gamma = 1.2$ is chosen to show the controller parameterization set and stability regions clearly. If we apply the same technique for $\gamma = 1.08$ the feasible region in $\mathbb{R}^3$

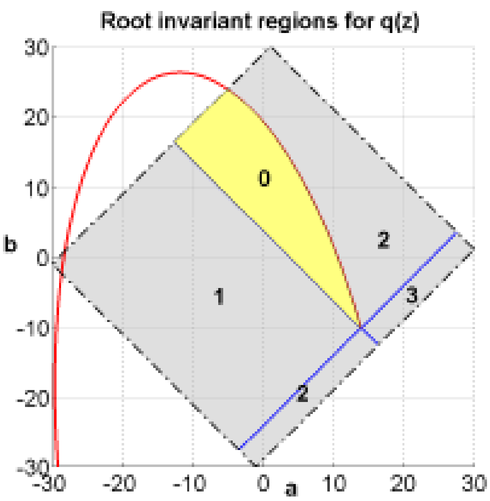

Fig. 3. Root invariant regions for $c = 30$.

shrinks, but we still get a solution:

$$F(s) = \frac{0.068s^3 + 3.77s^2 + 21.45s + 295.84}{9.93s^3 + 62.77s^2 + 187.25s + 296.27}.$$

It is easy to verify that

$$F(s_i) = \frac{\omega_i}{1.08}, \qquad \text{for} \quad i = 1, 2.$$

The function F is a unit with poles and zeros

$$\begin{aligned} \texttt{zero(F)} &= -50.9245, -2.2583 \pm j\ 8.9628 \\ \texttt{pole(F)} &= -3.3510, -1.4851 \pm j\ 2.5881 \end{aligned}$$

and from its Bode plot we find $\|F\|_\infty = \frac{295.84}{296.27} < 1$.

V. Conclusions

In this note we have modified the Nevanlinna-Pick interpolation problem appearing in the computation of the optimal strongly stabilizing controller minimizing the weighted sensitivity. By putting a bound on the norm of F^{-1}, a bound on the $\mathcal{H}^\infty$ norm of the controller can be obtained. We have obtained the optimal γ_{ss} as a function of ρ, where $\|F^{-1}\|_\infty \leq \rho$. The example illustrated that $\gamma_{ss}(\rho)$ converges to the optimal γ_{ss} for the unconstrained problem. The controller obtained here is again infinite dimensional, for practical purposes it needs to be approximated by a rational function. Another method for finding a low order F satisfying all the conditions is also illustrated with the given example. It searches for a first order free parameter leading to a unit f. Although it works for the example given here, in general this method can be quite conservative since the order of strongly stabilizing controllers for a given plant (even in the finite dimensional case) may have to be very large, [21].

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