Fixed-Order H-infinity Optimization of Time-Delay Systems

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1 Abstract

We consider the $\mathscr{L}_\infty$ or $\mathscr{H}_\infty$ norm optimization of the transfer function G,

$$G(j\omega,x)=C(x)\left(j\omega E-A_0(x)-\sum_{i=1}^{m}A_i(x)e^{-j\omega\tau_i}\right)^{-1}B(x)+D(x)e^{-j\omega\tau_0} \quad (1)$$

where the system matrices (A_i,B,C,D), $i=0,\ldots,m$ are real-valued, depending on fixed number of parameters $x\in\mathbb{R}^{n_x}$, and where the time delays, $(\tau_0,\ldots,\tau_m)$, are nonnegative real numbers.

This type of problems is encountered in control problems such as the $\mathscr{H}_\infty$ norm optimization or pseudospectral abscissa optimization of time-delay systems using fixed structure controllers.

Let the plant have the following state-space representation:

$$\begin{aligned}\dot{x}_p(t) &= A_0x_p(t)+\sum_{i=1}^{m}A_ix_p(t-\tau_i)+B_1w(t)+B_2u(t-\tau_0),\\ z(t) &= C_1x_p(t)+D_{11}w(t),\\ y(t) &= C_2x_p(t)+D_{22}u(t-\tau_0),\end{aligned} \quad (2)$$

and let the dynamic controller be represented by

$$\begin{aligned}\dot{x}_K(t) &= A_Kx_K(t)+B_Ky(t),\\ u(t) &= C_Kx_K(t)+D_Ky(t)\end{aligned} \quad (3)$$

where the system matrices are real valued with appropriate dimensions. The connection between the plant (2) and the controller (3) leads to a transfer function from w to z of the form (1) where the controller parameters x represents the elements of the matrices (A_K,B_K,C_K,D_K).

We optimize the $\mathscr{H}_\infty$ norm of G over the feasible design parameters in two steps:

- finding a point in the parameter space stabilizing the time-delay system G. This is done by minimizing the spectral abscissa of G. The resulting controller of the stabilization step is used as an initial point for the $\mathscr{H}_\infty$ optimization step,
- directly minimizing $\mathscr{H}_\infty$ norm over the parameter space. Since the algorithm is monotonically decreasing, it is guaranteed that the feasible set in the parameter space, where G is stable, is not left.

In both cases the objective function is nonsmooth, but differentiable almost everywhere. The optimization is done with the algorithm [1], which relies on the computation of the objective function and its gradient (where it exists).

This approach has been successfully applied in the finite dimensional case [2]. We present the extension of this approach to time-delay systems. The $\mathscr{H}_\infty$ optimization requires the $\mathscr{H}_\infty$ norm computation of G given the fixed parameters x. This computation is based on the connection between the singular values of a transfer function G and the imaginary axis eigenvalues of an infinite dimensional operator $\mathscr{L}_\xi$. Using a spectral method this operator is discretized, leading to an approximation of the $\mathscr{H}_\infty$ norm. Finally the approximate results are corrected by solving a set of equations which are obtained from the reformulation of the eigenvalue problem for $\mathscr{L}_\xi$ as a finite dimensional nonlinear eigenvalue problem.

The computation or optimization of pseudospectral abscissa can be done in a similar way.

References

[1] J. V. Burke, A. S. Lewis and M. L. Overton, "A robust gradient sampling algorithm for nonsmooth, nonconvex optimization, SIAM Journal on Optimization, vol.15, pp.751-779, 2005.

[2] S. Gumussoy and M. L. Overton, "Fixed-Order H-infinity Controller Design via HIFOO, a Specialized Nonsmooth Optimization Package," Proceedings of American Control Conference, pp. 2750-2754, 2008.